

Type 2 Projectile Motion Problems: Angled launch

A baseball player throws a ball to a teammate at 18m/s at an angle of 25° above the horizontal.

What are the initial horizontal and vertical velocities? What is the maximum height the ball reaches? How far away horizontally is the ball caught?

$$\begin{aligned} \sin 25 &= \frac{v_{oy}}{18} \\ \sin 25 \times 18 &= v_{oy} \\ 7.6071 \text{ m/s} & \end{aligned}$$

$$v_x = 16.3135 \text{ m/s}$$

X	Y
$v_x = 16.3135 \text{ m/s}$	$v_{oy} = 7.6071 \text{ m/s}$
	$a_y = -9.8 \text{ m/s}^2$
	$v_{fy} = 0$

Use  $v_f^2 = v_0^2 + 2ad$  to find max height

$$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ 0 & 7.6071 & -9.8 \end{array}$$

$$d = 2.9524 \text{ m}$$

$$\approx 3.0 \text{ m}$$

Since ball is caught at some height its final vertical velocity is -7.6071 m/s

$$v_f = v_0 + at \quad \text{to find } t$$

$$-7.6071 = 7.6071 + (-9.8)t$$

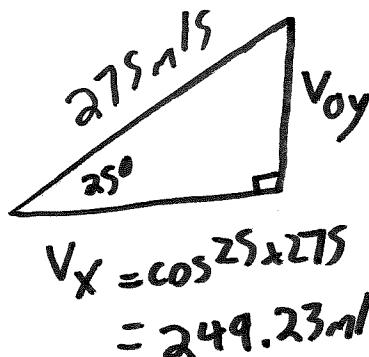
$$t = 1.5525 \text{ sec}$$

Use  $d = \bar{v}t$  to find  $d_x$

$$d = 25 \text{ m}$$

Problem: A cannonball is launched at 275 m/s at an angle of 25° above the horizontal.

What are the initial horizontal and vertical velocities? How long is the cannonball airborne? How far horizontally does the cannonball travel? What is the final velocity of the cannonball?



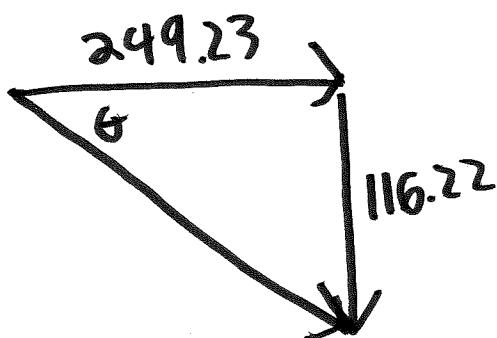
$$v_{oy} = \sin 25 \times 275 \\ = 116.22 \text{ m/s}$$

$$v_x = \cos 25 \times 275 \\ = 249.23 \text{ m/s}$$

x	y
$v_x = 249.23$	$v_{oy} = 116.22 \text{ m/s}$
	$a_y = -9.8 \text{ m/s}^2$
	$v_{fy} = -116.22 \text{ m/s}$

Use  $v_f = v_i + at$  to find  $t$   
 $t = 23.7184 \text{ sec}$   
 $\approx 24 \text{ sec}$

Use  $d = \bar{v} t$  to find  $d_x$   
 $d = 5911.33 \text{ m}$   
 $\approx 5900 \text{ m}$



$$\text{mag} = \sqrt{249.23^2 + 116.22^2} \\ = 274.996 \text{ m/s}$$

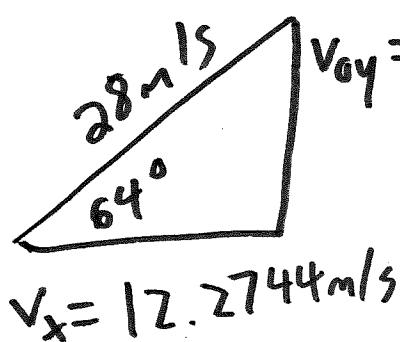
$$\theta = \tan^{-1}\left(\frac{116.22}{249.23}\right) \\ = 25^\circ$$

270 m/s, 25° below horizontal

Type 3 Projectile Motion Problems: Angled launch from a height

A cannonball is launched from a cliff 45m tall at 28 m/s an angle of  $64^\circ$  above the horizontal.

What are the initial horizontal and vertical velocities? What is the final velocity? How long is the cannonball airborne? What is the maximum height relative to the ground below?



$$v_{0y} = 25.166 \text{ m/s}$$

$$v_x = 12.2744 \text{ m/s}$$

x	y
$v_x = 12.2744$	$v_{0y} = 25.1662 \text{ m/s}$
	$a = -9.8 \text{ m/s}^2$
	$d_y = -45 \text{ m}$

①  $v_{fy} \rightarrow \text{use } v_f^2 = v_i^2 + 2ad$

$$v_f = \pm 38.9273 \text{ m/s}$$

$$v_f = -38.9273 \text{ m/s}$$

②  $\rightarrow v_f = v_i + at$   
 $t = 6.5 \text{ sec}$

③ Use  $v_{fy} = 0$  to find height  
 use  $v_f^2 = v_i^2 + 2ad$

$$d = 32 \text{ m}$$

$$32 + 45 = 77 \text{ m}$$

An archer standing on a castle 12 m tall shoots an arrow at a velocity of 68 m/s  $15^\circ$  above the horizontal. It hits a wall 200.0 m horizontally away.

At what height on the wall does the arrow hit?

$$\begin{aligned} & \text{Diagram: A right triangle with a hypotenuse of } 68 \text{ m/s at } 15^\circ \text{ to the horizontal.} \\ & \text{Vertical leg: } \sin 15 \times 68 = 17.5997 \\ & \text{Horizontal leg: } \cos 15 \times 68 = 65.6830 \end{aligned}$$

X	Y
$v_x = 65.6830 \text{ m/s}$	$v_{oy} = 17.5997 \text{ m/s}$
$d_x = 200.0 \text{ m}$	$a = -9.8 \text{ m/s}^2$
	$d_y = ?$

Use  $d = \bar{v}t$  to find  $t$

$$\begin{aligned} & \text{Diagram: A horizontal distance } 200 \text{ m is divided into segments by arrows pointing right. The first segment is labeled } 65.6830. \text{ To its right is the symbol } + \text{ followed by } 3.044. \\ & + = 3.044 \end{aligned}$$

Use  $d = v_0 t + \frac{1}{2} a t^2$  to find  $d_y$

$$d = 8.17 \text{ m above initial}$$

Hits wall at  $12 + 8.17 \text{ m}$

$$\approx 20 \text{ m} \approx 2.0 \times 10^1 \text{ m}$$